Lesson Plan: Discovering Relationships in Recursion Through the Tower of Hanoi

Objective: Students will explore and discover the recursive relationship in the Tower of Hanoi problem through collaborative investigation and pattern recognition.

Materials Needed:

- Tower of Hanoi Applet:
- Whiteboard or flip chart
- Markers
- Paper for each group to record observations

Lesson Outline:

1. Introduction
    - Briefly introduce the Tower of Hanoi problem and its rules, emphasizing that they will explore how the number of moves relates to the number of rings.
    - Pose an open-ended question: “What do you think will happen to the number of moves as we increase the number of rings? Why?”
2. Group Activity Setup
    - Divide the class into small groups (3-4 students per group).
    - Assign each group a different number of rings (Group A: 2 rings, Group B: 3 rings, Group C: 4 rings, Group D: 5 rings).
    - Explain that their goal is to find out how many moves it takes to solve their assigned number of rings.
3. Hands-On Exploration
    - Instruct each group to use the Tower of Hanoi applet to solve their assigned number of rings.
    - As they work, they should:
        - Count the number of moves required to solve their puzzle.
        - Discuss among themselves and make predictions about the next number of rings.

- Record their observations, including any initial guesses, calculations, and the actual number of moves.

- Encourage them to think critically about any patterns they notice without leading them directly to the formula.

4. Group Comparison and Discussion

- Have each group present their findings, focusing on:

- The number of rings they worked with.

- The number of moves it took to solve the puzzle.

- Any patterns or relationships they discovered.

- On the whiteboard, list the results from all groups and ask guiding questions:

- “What do you notice about how the number of moves changes as we increase the number of rings?”

- “Can we come up with a general way to describe this relationship?”

- Encourage students to hypothesize about the mathematical relationship based on the observed data.

5. Conclusion

- Summarize the discussions, highlighting any patterns students noticed (e.g., if they identified a doubling pattern).

- Introduce the formula 2^n-1only after they’ve explored the relationships. Ask if they can derive it from their observations.

Assessment:

- Observe group discussions for engagement and critical thinking.

- Collect papers to review the observations and relationships noted by each group.

Extensions (if time allows):

- Challenge groups to calculate and predict the moves required for larger numbers of rings using their discovered relationship.

- Discuss how this concept of recursion might apply in other areas, inviting students to provide examples from mathematics or computer science.